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ZAMONAVIY INNOVATSION TADQIQOTLARNING DOLZARB MUAMMOLARI VA RIVOJLANISH TENDENSIYALARI: YECHIMLAR VA ISTIQBOLLAR

mavzusidagi Respublika ilmiy-texnik anjuman materiallari to‘plami

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Respublika miqyosidagi ilmiy-texnik anjuman materiallarida zamonaviy innovatsion tadqiqotlarning dolzarb muammolari va rivojlanish tendensiyalari sohasidagi innovatsion tadqiqotlar aks etgan.

Globallashuv sharoitida davlatimizni yanada barqaror va jadal sur’atlar bilan rivojlantirish bo‘yicha amalga oshirilayotgan islohotlar samarasini yaxshilash sohasidagi ilmiy-tadqiqot ishlariga alohida e’tibor qaratilgan. Zero iqtisodiyotning, ijtimoiy sohalarni qamrab olgan modernizatsiya jarayonlari, hayotning barcha sohalarini liberallashtirishni talab qilmoqda.

Ushbu ilmiy ma’ruza tezislari to‘plamida mamlakatimiz va xorijlik turli yo‘nalishlarda faoliyat olib borayotgan mutaxassislar, olimlar, professor-o‘qituvchilar, ilmiy tadqiqot institutlari va markazlarining ilmiy xodimlari, tadqiqotchilari, magistr va talabalarning ilmiy-tadqiqot ishlari natijalari mujassamlashgan.

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Mazkur to‘plamga kiritilgan ma’ruza tezislarining mazmuni, undagi statistik ma’lumotlar va me’yoriy hujjatlarning to‘g‘riligi hamda tanqidiy fikr-mulohazalar, keltirilgan takliflarga mualliflarning o‘zlari mas’uldirlar.

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5. Otamuratov R., Muhammadiyev A. YATROGENIYA KELIB CHIQISH TARIXI VA ILK QARASHLAR //Журнал Педагогики и психологии в современном образовании. – 2024. – Т. 4. – №. 2. – С. 3-9.

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ASYMPTOTICS OF EIGENVALUES OF RANK-ONE PERTURBATIONS OPERATOR ON LATTICE

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Abstract. We consider the family $H_v(\mu)$ of discrete Schrödinger-type operator in two-dimensional lattice $\mathbb{T}^2$. We establish the existence or non-existence and also the finiteness of eigenvalues of $H_v(\mu)$ lying above the essential spectrum. Moreover, we study the properties of eigenvalues as a function of μ, in particular, we find the asymptotics of eigenvalues as sufficiently small and positive μ.

Keywords: operator, spectrum, eigenvalue, eigenvalue asymptotic.

The family $H_v(\mu)$ given by below can be seen as a discrete counterpart of a continuous nonlocal Schrödinger-type operators appearing as a diffusion generator in

certain stochastic models (see e.g. [2–6]). At the same time, they can also be seen as an effective one-particle discrete Schrödinger operator $H_v(\mu; K)$ parametrized by quasi-momenta $K \in T\,d,$ associated to the Hamiltonian of a system of two arbitrary and/or identical particles in *d*-dimensional lattice Z^d (see e.g. [4–11]).

Such lattice models have become popular in recent years because they represent a minimal, natural Hamiltonian describing the systems of ultracold atoms in optical lattices – systems with highly controllable parameters such as lattice geometry and dimensionality, particle masses, tunneling, temperature etc. (see e.g., [7,8,9,10] and references therein).

In the momentum space representation, the operator acts in $L^2(\mathbb{T}^2)$ by

$$H_v(\mu) := H_0 + \mu V_v, \quad \mu > 0.$$

The free Hamiltonian H_0 is the multiplication operator in $L^2(\mathbb{T}^2)$ by the function

$$\varepsilon := 2\pi\mathcal{F}\hat{\varepsilon}$$

so-called the *dispersion relation* of the particle and the potential $V_{a,b}$ acts on $L^2(\mathbb{T}^2)$ as a convolution operator

$$V_v f(p) = v(p)\int_{\mathbb{T}^2} v(q)f(q)\mathrm{d}q$$

defined in $L^2(\mathbb{T}^2),$ where $v(\cdot)$ is a given nonzero real-analytic function on $\mathbb{T}^2$. By the clasic Weyl theorem we obtain

$$\sigma_{\mathrm{ess}}\big(H_v(\mu)\big) = \sigma(H_0) = [\varepsilon_{\min}, \varepsilon_{\max}], \qquad \mu > 0,$$

where

$$\varepsilon_{\min} := \min \varepsilon, \quad \varepsilon_{\max} := \max \varepsilon.$$

In what follows we always assume:

Hypothesis 1 *The dispersion relation* ε *is a real-valued even function, symmetric with respect to coordinate permutations and having a non-degenerate unique maximum at* $\vec{\pi} = (\pi, \pi) \in \mathbb{T}^2$. *Moreover,* ε *is analytic near* $\vec{\pi}$.

In this paperwe study the eigenvalues of operators by the min-max principle, $H_v(\mu)$ can have at most one eigenvalue outside the essential spectrum. Also note that $z_0 > \varepsilon_{\max}$ is eigenvalue of $H_v(\mu)$ if and only if z_0 is a zero of the corresponding Fredholm determinant

$$\Delta_v(\mu; z) := 1 - \mu\int_{\mathbb{T}^2} \frac{v(q)^2\mathrm{d}q}{z - \varepsilon(q)}.$$

Such an equivalence will be frequently used subsequently. First we study the existence of eigenvalues of $H_v(\mu).$

Theorem 1. *Let* v *be nonzero real-analytic function on* $\mathbb{T}^2$ *and let*

$$\mu_v^0 := \left[\int_{\mathbb{T}^2} \frac{v(q)^2 dq}{\varepsilon_{max} - \varepsilon(q)}\right]^{-1}.$$

Then $\mu_v^0 = 0$ *if* $v(\vec{\pi}) \neq 0$ *and* $\mu_v^0 \in (0, +\infty)$ *if* $v(\vec{\pi}) = 0$. *Moreover, for any* $\mu > \mu_v^0$ *the operator* $H_v(\mu)$ *has a unique eigenvalue* $z_v(\mu) > \varepsilon_{max}$ *and the corresponding eigenfunction is*

$$f_\mu(p) = \frac{v(p)}{z_v(\mu) - \varepsilon(p)}.$$

Moreover, the function $z_v(\cdot)$ *is real-analytic, strictly increasing and strictly convex in* $(\mu_v^0, +\infty)$ *with the asymptotics*

$$z_v(\mu) \searrow \varepsilon_{max} \quad as \quad \mu \searrow \mu_v^0, \qquad (1)$$

and

$$z_v(\mu) = \mu \int_{\mathbb{T}^2} v(q)^2\, dq + \left[\int_{\mathbb{T}^2} v(q)^2\, dq\right]^{-1} \int_{\mathbb{T}^2} v(q)^2\, \varepsilon(q) dq + O\left(\frac{1}{\mu}\right)$$

as $\mu \to +\infty$. *Finally, if* $v \in L^{2,\omega}(\mathbb{T}^2)$ *for some* $\omega \in \{os, oa, ea, es\}$, *then* $L^{2,\omega}(\mathbb{T}^2)$ *is invariant subspace of* $H_v(\mu)$ *and* $f_\mu \in L^{2,\omega}(\mathbb{T}^2)$.

We drop the proof since the all assertions but the last one can be done along the lines of [1, Theorem 2.1] using Proposition 1. The last assertion is obvious.

Next we study the asymptotics of $z_v(\mu)$ as $\mu \searrow \mu_v^0$.

Theorem 2.

(a) Assume that $v(\vec{\pi}) \neq 0$. *Then for sufficiently small and positive* $\mu > 0$

$$z_v(\mu) = \varepsilon_{max} + c_v\, e^{-\frac{1}{4\pi^2 J_0 v(\vec{\pi})^2\, \mu}} + \sum_{\substack{n,m\geq 0,\\ n+m\geq 1}} c_{nm} \mu^n \left(\frac{1}{\mu}\, e^{-\frac{1}{4\pi^2 J_0 v(\vec{\pi})^2\, \mu}}\right)^{m+1}, \quad (2)$$

where $J_0 > 0$ *is given by Remark 3.1,* $\{c_{nm}\}$ *are real coefficients,*

$c_v := e^{\frac{\omega_\varepsilon}{4\pi^2 J_0 v(\vec{\pi})^2}}$

and $\omega_\varepsilon \in \mathbb{R}$ *is a constant depending only on* ε;

(b) Assume that $v(\vec{\pi}) = 0$ *and* $\nabla v(\vec{\pi}) \neq 0$. *Then for sufficiently small and positive* $\mu - \mu_v^0$

$$z_v(\mu) = \varepsilon_{max} + \frac{c_v\, (\mu - \mu_v^0)}{-ln(\mu - \mu_v^0)} + \sum_{\substack{n,m,k\geq 0,\\ n+m+k\geq 1}} c_{nmk} (\mu - \mu_v^0)^n \left(\frac{\mu - \mu_v^0}{-ln(\mu - \mu_v^0)}\right)^{m+1} \left(\frac{lnln(\mu - \mu_v^0)^{-1}}{-ln(\mu - \mu_v^0)}\right)^k, \quad (3)$$

where $\{c_{nmk}\}$ *are real coefficients and*

$$c_v = \frac{1}{2\pi^2 J_0 (\mu_v^0)^2} \left[\left(\frac{\partial v}{\partial q_1} (\vec{\pi}) \right)^2 \left(\frac{\partial \psi}{\partial y_1} (\vec{0}) \right)^2 + \left(\frac{\partial v}{\partial q_2} (\vec{\pi}) \right)^2 \left(\frac{\partial \psi}{\partial y_2} (\vec{0}) \right)^2 \right]^{-1};$$

(c) Assume that $v(\vec{\pi}) = 0$ *and* $\nabla v(\vec{\pi}) = 0.$ *Then for sufficiently small and positive* $\mu - \mu_v^0$

$$z_v(\mu) = \varepsilon_{max} + c_v (\mu - \mu_v^0) + \sum_{\substack{n,m \geq 0, \\ n+m \geq 1}} c_{nm} (\mu - \mu_v^0)^{n+1} (-(\mu - \mu_v^0) ln(\mu - \mu_v^0))^m, \qquad (3^*)$$

where $\{c_{nm}\}$ *are real coefficients and*

$$c_v = \frac{1}{(\mu_v^0)^2} \left[\int_{\mathbb{T}^2} \frac{v(q)^2 \, dq}{\left(\varepsilon_{max} - \varepsilon(q) \right)^2} \right]^{-1} > 0.$$

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EXPANSION OF EIGENVALUES OF SCHRÖDINGER-TYPE OPERATORS ON LATTICES

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Abstract. We study the discrete spectrum of a wide class of discrete Schrödinger-type operators $\widehat{H}_\mu = \widehat{H}_0 + \mu\widehat{V}$ on the two-dimensional lattice $\mathbb{Z}^2$, where $\widehat{H}_0$ is a Laurent-Toeplitz-type convolution operator, $\hat{V}$ is a finite rank potential and $\mu \geq 0$ is the coupling constant. In the case of existence, we study the asymptotic of eigenvalues of $\widehat{H}_\mu$ as $\mu \to +\infty$.

Keywords and phrases: one-particle discrete Schrödinger-type operators, potential, essential spectrum, eigenvalue, asymptotic behavior.

In the present paper we replace the Euclidean $d-$dimensional space $\mathbb{R}^d$ by the two-dimensional lattice $\mathbb{Z}^2$. Let $\ell^2(\mathbb{Z}^2)$ be the Hilbert space of square-summable functions on $\mathbb{Z}^2$. We denote by

$$\mathbb{T}^2 = (\mathbb{R}/2\pi\mathbb{Z})^2 = (-\pi, \pi]^2$$

be the two – dimensional torus, the dual group to $\mathbb{Z}^2$ and $L^2(\mathbb{T}^2)$ be the Hilbert space of square-integrable functions defined on $\mathbb{T}^2$. Elements of $L^2(\mathbb{T}^2)$ can be considered as functions $f: \mathbb{R}^d \to \mathbb{C}$, in periodic each coordinates.

In the coordinate space representation, the energy operator $\widehat{H}_\mu$ of a one-particle system on the two-dimensional lattice $\mathbb{Z}^2$ with a potential field $\hat{v}$ is defined as

$$\widehat{H}_\mu := \widehat{H}_0 + \mu\widehat{V} \tag{1}$$

where the free energy operator H_0 is a Laurent-Toeplitz-type operator in $\ell^2(\mathbb{Z}^2)$