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ZAMONAVIY INNOVATSION TADQIQOTLARNING DOLZARB MUAMMOLARI VA RIVOJLANISH TENDENSIYALARI: YECHIMLAR VA ISTIQBOLLAR

mavzusidagi Respublika ilmiy-texnik anjuman materiallari to‘plami

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“Zamonaviy innovatsion tadqiqotlarning dolzarb muammolari va rivojlanish tendensiyalari: yechimlar va istiqbollar” mavzusidagi Respublika ilmiy-texnik anjuman materiallari to‘plami – Jizzax: O‘zMU Jizzax filiali, 2024-yil 17-18-may. 410-bet.

Respublika miqyosidagi ilmiy-texnik anjuman materiallarida zamonaviy innovatsion tadqiqotlarning dolzarb muammolari va rivojlanish tendensiyalari sohasidagi innovatsion tadqiqotlar aks etgan.

Globallashuv sharoitida davlatimizni yanada barqaror va jadal sur’atlar bilan rivojlantirish bo‘yicha amalga oshirilayotgan islohotlar samarasini yaxshilash sohasidagi ilmiy-tadqiqot ishlariga alohida e’tibor qaratilgan. Zero iqtisodiyotning, ijtimoiy sohalarni qamrab olgan modernizatsiya jarayonlari, hayotning barcha sohalarini liberallashtirishni talab qilmoqda.

Ushbu ilmiy ma’ruza tezislari to‘plamida mamlakatimiz va xorijlik turli yo‘nalishlarda faoliyat olib borayotgan mutaxassislar, olimlar, professor-o‘qituvchilar, ilmiy tadqiqot institutlari va markazlarining ilmiy xodimlari, tadqiqotchilari, magistr va talabalarning ilmiy-tadqiqot ishlari natijalari mujassamlashgan.

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Mazkur to‘plamga kiritilgan ma’ruza tezislarining mazmuni, undagi statistik ma’lumotlar va me’yoriy hujjatlarning to‘g‘riligi hamda tanqidiy fikr-mulohazalar, keltirilgan takliflarga mualliflarning o‘zlari mas’uldirlar.

[10] F.Almuratov: Ikki o‘lchamli qo‘zg‘alishli model operatorining diskret spektri Информатика и инженерные технологии 1 (2), 235-238

[11] M Pardabaev, F Almuratov: On the Spectrum of the Discrete Bilaplacian with Rank-One Perturbation, Geometry, Integrability and Quantization, Papers and Lecture Series 26, 39-52

EXPANSION OF EIGENVALUES OF SCHRÖDINGER-TYPE OPERATORS ON LATTICES

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Abstract. We study the discrete spectrum of a wide class of discrete Schrödinger-type operators $\widehat{H}_\mu = \widehat{H}_0 + \mu\widehat{V}$ on the two-dimensional lattice $\mathbb{Z}^2$, where $\widehat{H}_0$ is a Laurent-Toeplitz-type convolution operator, $\hat{V}$ is a finite rank potential and $\mu \geq 0$ is the coupling constant. In the case of existence, we study the asymptotic of eigenvalues of $\widehat{H}_\mu$ as $\mu \to +\infty$.

Keywords and phrases: one-particle discrete Schrödinger-type operators, potential, essential spectrum, eigenvalue, asymptotic behavior.

In the present paper we replace the Euclidean $d-$dimensional space $\mathbb{R}^d$ by the two-dimensional lattice $\mathbb{Z}^2$. Let $\ell^2(\mathbb{Z}^2)$ be the Hilbert space of square-summable functions on $\mathbb{Z}^2$. We denote by

$$\mathbb{T}^2 = (\mathbb{R}/2\pi\mathbb{Z})^2 = (-\pi, \pi]^2$$

be the two – dimensional torus, the dual group to $\mathbb{Z}^2$ and $L^2(\mathbb{T}^2)$ be the Hilbert space of square-integrable functions defined on $\mathbb{T}^2$. Elements of $L^2(\mathbb{T}^2)$ can be considered as functions $f: \mathbb{R}^d \to \mathbb{C}$, in periodic each coordinates.

In the coordinate space representation, the energy operator $\widehat{H}_\mu$ of a one-particle system on the two-dimensional lattice $\mathbb{Z}^2$ with a potential field $\hat{v}$ is defined as

$$\widehat{H}_\mu := \widehat{H}_0 + \mu\widehat{V} \tag{1}$$

where the free energy operator H_0 is a Laurent-Toeplitz-type operator in $\ell^2(\mathbb{Z}^2)$

$$\widehat{H}_0\hat{f}(x) = \sum_{y\in\mathbb{Z}^2} \hat{\varepsilon}(x-y)\hat{f}(y), \qquad \hat{f} \in \ell^2(\mathbb{Z}^2),$$

given by given by a *Hopping matrix* be $\hat{\varepsilon} \in \ell^1(\mathbb{Z}^2)$ of the particle which satisfies $\hat{\varepsilon}(x) = \overline{\hat{\varepsilon}(-x)}$ for all $x \in \mathbb{Z}^2$, and the potential energy operator is the multiplication in $\ell^2(\mathbb{Z}^2)$ by the function

$$\hat{v}(x) = \begin{cases} a, & if \quad x = 0, \\ b, & if \quad |x| = 1, \\ 0 & if \quad |x| > 1, \end{cases}$$

where $a, b \in \mathbb{R} \setminus \{0\}$.

In the momentum space representation, the operator acts in $L^2(\mathbb{T}^2)$ by

$H_\mu = \mathcal{F}\widehat{H}_0\mathcal{F}^* + \mu\mathcal{F}\hat{V}\mathcal{F}^* = H_0 + \mu V,$

where

$$\mathcal{F}: \ell^2(\mathbb{Z}^2) \to L^2(\mathbb{T}^2), \quad \mathcal{F}\hat{f}(p) = (2\pi)^{-1} \sum_{x\in\mathbb{Z}^2} \hat{f}(x)e^{\mathrm{i}(p,x)}$$

is the standard Fourier transform with the inverse. The free Hamiltonian H_0 is the multiplication operator in $L^2(\mathbb{T}^2)$ by the function $\varepsilon := 2\pi\mathcal{F}\hat{\varepsilon}$ socalled the *dispersion relation* of the particle and the potential V acts on $L^2(\mathbb{T}^2)$ as a convolution operator

$$Vf(p) = \frac{1}{2\pi}\int_{\mathbb{T}^2} v(p-q)f(q)dq$$

with the analytic on $\mathbb{T}^2$ kernel

$$v(p) = \mathcal{F}\hat{v}(p) = \frac{1}{2\pi}\sum_{x\in\mathbb{Z}^2} \hat{v}(x)e^{ip\cdot x} = \frac{1}{2\pi}\left(a + 2b\sum_{i=1}^{2}\cos p_i\right).$$

Since V is compact, by Weyl's Theorem [8, Theorem XIII.14],

$\sigma_{ess}(H_\mu) = \sigma(H_0) = [\min \varepsilon, \max \varepsilon] = [\varepsilon_{min}, \varepsilon_{max}]$

In what follows we always assume:

Hypothesis 1. *The dispersion relation* ε *is a real-valued even function, symmetric with respect to coordinate permutations and having a non-degenerate unique maximum at* $\vec{\pi} = (\pi, \pi) \in \mathbb{T}^2$. *Moreover,* ε *is analytic near* $\vec{\pi}$.

Recall that

$L^2(\mathbb{T}^2) = L^{2,es}(\mathbb{T}^2) \oplus L^{2,ea}(\mathbb{T}^2) \oplus L^{2,os}(\mathbb{T}^2) \otimes L^{2,oa}(\mathbb{T}^2).$

Recalling that

$$Vf(p) = \frac{a}{4\pi^2}\int_{\mathbb{T}^2} f(q)\mathrm{d}q + \frac{b}{4\pi^2}\int_{\mathbb{T}^2} (2\cos p_1\cos q_1 + 2\cos p_2\cos q_2)\, f(q)\, \mathrm{d}q$$

$$+\frac{b}{4\pi^2}\int_{\mathbb{T}^2}(2\sin p_1\sin q_1+2\sin p_2\sin q_2)\,f(q)\,dq$$

we get that each $L^{2,\omega}$ is invariant also with respect to V.

Thus, we study the discrete spectrum of H_μ separately restricted to these subspaces. Moreover, we study the asymptotics of eigenvalues as $\mu \to +\infty$. The existence of eigenvalues for this operator is explored in this work [2, 5, 7-11]. In this work, we only define the asymptotic expansions of eigenvalues as $\mu \to +\infty$.

Next, we study the asymptotics of eigenvalues of H_μ as $\mu \to +\infty$.

Theorem 3. *(a) Let* $\omega \in \{os, oa, ea\}$ *and let* $E_\omega(\cdot)$ *be the unique eigenvalue of* $H_\mu|_{L^{2,\omega}(\mathbb{T}^2)}$. *Then*

$$E_\omega(\mu)=\begin{cases} b\mu+\dfrac{1}{4\pi^2}\displaystyle\int_{\mathbb{T}^2}(\sin q_1+\sin q_2)^2\varepsilon(q)\,dq+O(1/\mu) & if\ \ \omega=os,\\ b\mu+\dfrac{1}{4\pi^2}\displaystyle\int_{\mathbb{T}^2}(\sin q_1-\sin q_2)^2\varepsilon(q)\,dq+O(1/\mu) & if\ \ \omega=oa,\\ b\mu+\dfrac{1}{4\pi^2}\displaystyle\int_{\mathbb{T}^2}(\cos q_1-\cos q_2)^2\varepsilon(q)\,dq+O(1/\mu) & if\ \ \omega=ea \end{cases} \quad (2)$$

as $\mu \to +\infty$.

(b) Let $\omega = es, ab < 0$ *and let* $E_{es}(\cdot)$ *be the unique eigenvalue of* $H_\mu|_{L^{2,es}(\mathbb{T}^2)}$. *Then*

$$E_{es}(\mu)= = \begin{cases} b\mu+\dfrac{b}{4\pi^2}\displaystyle\int_{\mathbb{T}^2}(\cos q_1+\cos q_2)^2\varepsilon(q)\,dq+O\left(\frac{1}{\mu}\right) & if\ \ b>0>a\\ a\mu+\dfrac{a}{4\pi^2}\displaystyle\int_{\mathbb{T}^2}\varepsilon(q)\,dq+O\left(\frac{1}{\mu}\right) & if\ \ a>0>b \end{cases} \quad (3)$$

as $\mu \to +\infty$.

(c) Let $\omega = es,\ a, b > 0$ *and let* $E_{es}^{(1)}(\cdot)$ *and* $E_{es}^{(2)}(\cdot)$ *be the eigenvalues of* $H_\mu|_{L^{2,es}(\mathbb{T}^2)}$. *Then*

$$\max\left\{E_{es}^{(1)}(\mu),E_{es}^{(2)}(\mu)\right\}=\max\{\psi_a(\mu),\psi_b(\mu)\}+O\left(\frac{1}{\mu}\right) \quad (4)$$

and

$$\min\left\{E_{es}^{(1)}(\mu),E_{es}^{(2)}(\mu)\right\}=\min\{\psi_a(\mu),\psi_b(\mu)\}+O\left(\frac{1}{\mu}\right) \quad (5)$$

as $\mu \to +\infty$, *where*

$$\psi_a(\mu) := a\mu+\frac{a}{4\pi^2}\int_{\mathbb{T}^2}\varepsilon(q)\,dq,$$

$$\psi_b(\mu) := b\mu + \frac{b}{4\pi^2}\int_{\mathbb{T}^2}(\cos q_1 + \cos q_2)^2\varepsilon(q)\,dq.$$

References:

[1] Sergio Albeverio, S. Lakaev, Z. Muminov: Schrodinger Operators on Lattices. The Efimov Effect and¨ Discrete Spectrum Asymptotics. Ann. Henri Poincar. 5 (2004).

[2] A.T.Boltaev, F.M.Almuratov: The Existence and Asymptotics of Eigenvalues of Schrodinger Operator on Two Dimensional Lattices. Lobachevskii J. Math. 43:12, 3460–3470 (2022)

[3] M. Klaus: On the bound states of Schrodinger operators in one dimension. Ann. Phys.¨ 108 (1977), 288-300.

[4] M. Klaus, B. Simon: Coupling constant thresholds in nonrelativistic Quantum Mechanics. I. Short-range two body case. Ann. Phys. 130 (1980), 251–281.

[5] Sh. Kholmatov, S. Lakaev And F. Almuratov: On the spectrum of Schrodinger-type operators on two dimensional lattices. J. Math. Anal. Appl. 126363 (2022).

[6] Sh. Kholmatov And M. Pardabaev: On spectrum of the discrete bilaplacian with zero-range perturbation. Lobachevskii J. Math. 42, 1286–1293 (2021).

[7] S.N. Lakaev, A.T. Boltaev And F.M. Almuratov: On The discrete spectra of Schrodinger-type operators on one dimensional lattices, Lobachevskii J. Math. 43:3, 1523—1536 (2022).

[8] M. Reed, B. Simon: Modern Methods of Mathematical Physics. IV: Analysis of Operators. Academic Press, New York, 1978.

[9] B. Simon: The bound state of weakly coupled Schrodinger operators in one and two dimensions. Ann. Phys. 97 (1976), 279–288.

[10] F.Almuratov: Ikki o‘lchamli qo‘zg‘alishli model operatorining diskret spektri Информатика и инженерные технологии 1 (2), 235-238

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Annotatsiya. Ushbu maqolada zanjirli kasrlar haqida ma’lumotlar berilib shu zanjirli kasr yordamida diofont tenglamalarini yechishda foydalanganmiz. Qisqa qilib aytganda diofant tenglamalarini yechishda chekli uzluksiz kasrga yoyish orqali